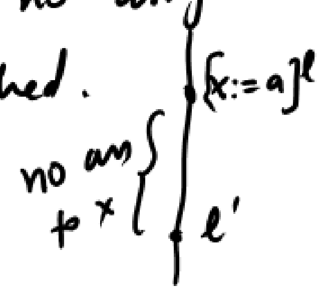
Dataflow Analysis

– Feb 19

```
y := x ; 1
z := 1 2
while [y>1] 3 do (
        z = z * y 4
        y := y - 1 5
)
y := 0 6
```

Reaching Definitions.

An assignment $[x := a]^l$ reaches entry or exit of l' if there is an execution where $[x := a]^l$ occurs and after that there is no assignment to x till l' is reached.



RD is undecidable.

$$[x := a]^{\ell} \ ; \ S \ ; \ [y := x]^{\ell'}$$

Programming Language.

$$a ::= x \mid n \mid a_1 \ op_a \ a_2$$

$$b ::= \text{true} \mid \text{false} \mid a_1 \ op_r \ a_2 \mid b_1 \ op_b \ b_2 \mid op_b' \ b$$

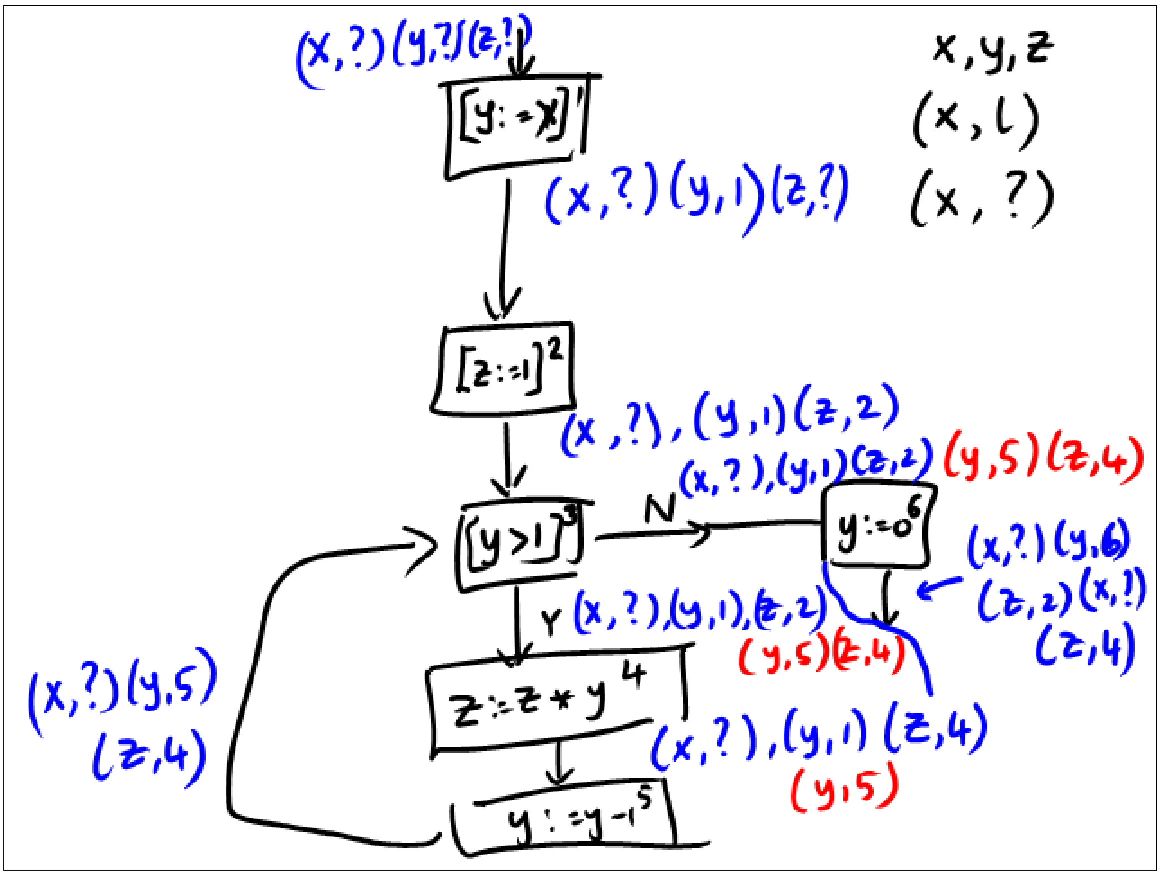
Eg. $op_a = \{ +, -, \ldots \}$

$op_r = \{ <, =, >, \leq \ldots \}$

$op_b = \{ \wedge, \vee,$ [illegible] $\}$

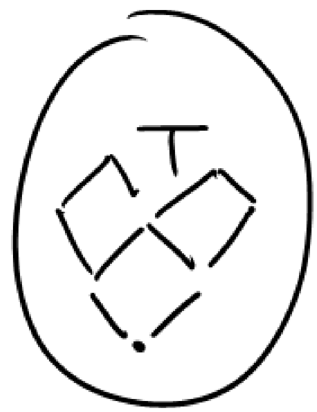
Statements

$$S ::= [x := a]^{\ell} \mid [\text{skip}]^{\ell} \mid$$
$$\text{if } [b]^{\ell} \text{ then } S_1 \text{ else } S_2 \mid$$
$$\text{while } [b]^{\ell} \text{ do } S \mid$$
$$S_1 ; S_2$$


x, y, z
(x, l)
(x, ?)
(x,?)(y,?)(z,?)
[y:=x]1
(x,?)(y,1)(z,?)
[z:=1]2
(x,?),(y,1)(z,2)
(x,?),(y,1)(z,2) (y,5)(z,4)
[y>1]3
N
y:=0 6
(x,?)(y,6)
(z,2)(x,?)
(z,4)
Y (x,?),(y,1),(z,2)
(y,5)(z,4)
z:=z*y 4
(x,?),(y,1) (z,4)
(y,5)
y:=y-1 5
(x,?)(y,5)
(z,4)

In general,

if $f : T \longrightarrow T$



RD analysis

$$f : T \longrightarrow T$$

$$T = \langle DF'_{entry}, DF'_{exit}, \ldots\ldots \rangle$$

$T = \langle DF^i_{entry}, DF^i_{exit} \rangle_{i=1,\dots 6}$

Lattice:

$\langle DF^i_{entry}, DF^i_{exit} \rangle \leq \langle DF'^i_{entry}, DF'^i_{exit} \rangle$

iff $\forall i.\ DF^i_{entry} \subseteq DF'^i_{entry}$

$DF^i_{exit} \subseteq DF'^i_{exit}$

$f : T \to T$ computing RD is monotonic.

So there is a lfp.

Hence RD : lfp f

Monotonic does not say
$X \subseteq f(X)$.

but says that
$X \subseteq Y \Rightarrow f(X) \subseteq f(Y)$