


Meet-over-paths
analysis using
automata.

Interprocedural flow analysis
using pushdown automata

Fix D - a set of facts (finite)

$$f_s : 2^D \rightarrow 2^D$$

Program P .

$$\text{MOP}(n) = \prod_{\substack{\pi = n_0 n_1 \dots n_k \\ n_k = n}} f_{\pi}(\emptyset)$$


$$f_{n_0 \dots n_k}(s) = f_{n_k}(\dots f_{n_1}(f_{n_0}(s)))$$

Program $P \equiv$:

Loc: $\{P_1, P_2, \dots, P_n\}$

Edge: $\{P \times S \times P\}$

Dataflow facts: D , Flow functions
 $\{f_s \mid s \in S\}$
 $f_s: 2^D \rightarrow 2^D$

Automaton . $Q = P \times 2^D$

$(P, X) \rightarrow (P', X')$

if $\exists s \in S. P \xrightarrow{s} P' \wedge$
 $X' = f_s(X) \dots$

Initial state: (P_{init}, \emptyset)

$d \in MOP(n)$ iff

a state in

$\{(P_n, X) \mid d \in X\}$

is reachable.

}
...
d
...
n

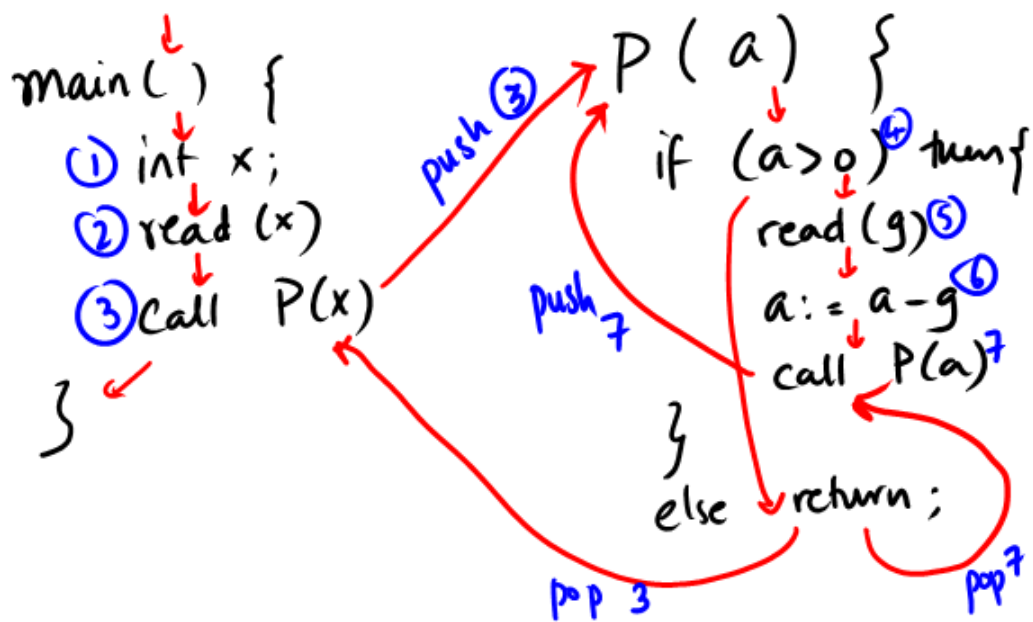
OR

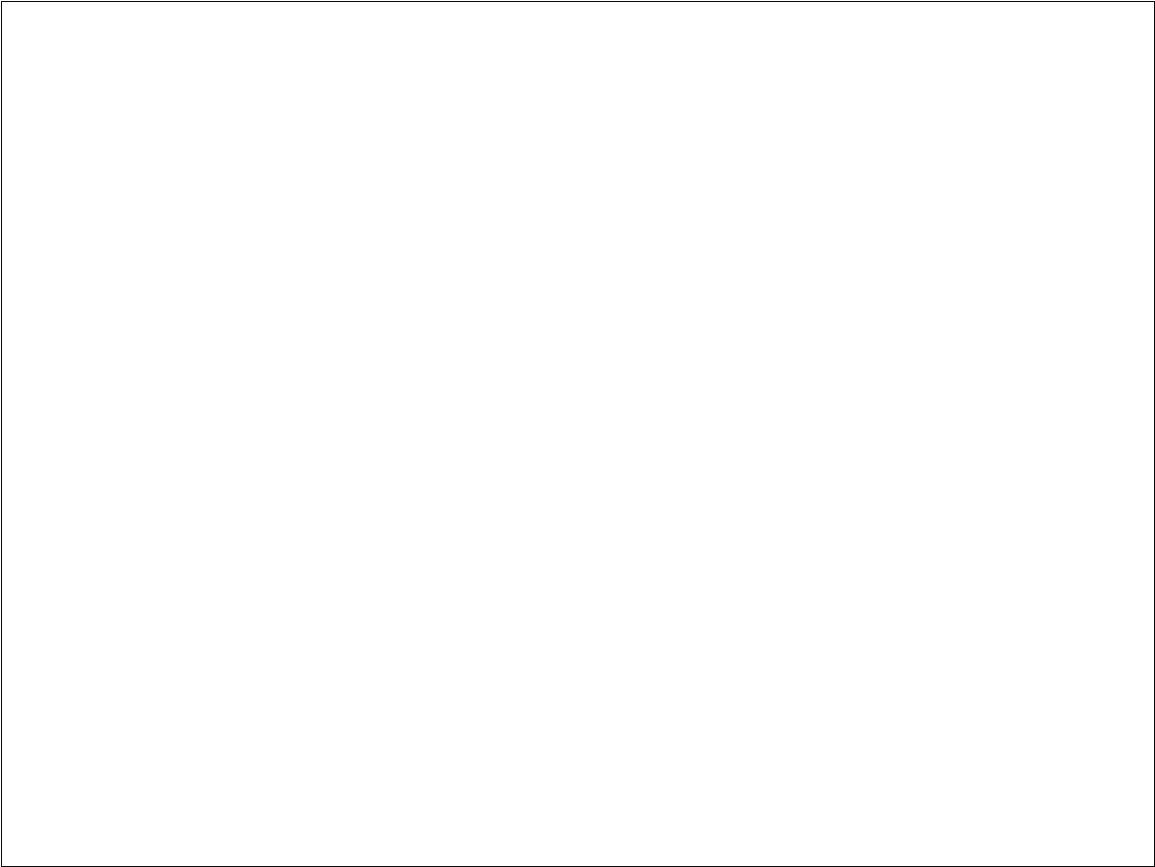
Make

$F = \{(P_n, X) \mid d \in X\}$

Is language of automaton empty?

Interprocedural flows.





Pushdown automata model Interprocedural control flow.

- At a call, push current pc.
- At a return, pop pc and
goto pc.
- Calls & Returns are deterministic.
- Captures control flow accurately.

Pushdown automata. Σ

$$A = (Q, q_0, \delta, \Gamma, F)$$

finite set
of states

finite
alphabet

$F \subseteq Q$

$(q, a, b, c, q') \in \delta$
 $q \xrightarrow{a, \text{pop } b, \text{push } c} q'$

$$\delta \subseteq Q \times \Sigma_{\epsilon} \times \Gamma_{\epsilon} \times \Gamma_{\epsilon} \times Q$$

$$\Sigma_{\epsilon} = \Sigma \cup \{\epsilon\}$$
$$\Gamma_{\epsilon} = \Gamma \cup \{\epsilon\}$$

Config: (q, σ) $q \in Q$
 $\sigma \in \Gamma^*$.

Moves:

$(q, \sigma) \xrightarrow{a} (q', \sigma')$

if $\exists (q, a, b, c, q') \in \delta$

s.t. $\sigma = b \sigma_1$ for some $\sigma_1 \in \Gamma^*$

$\sigma' = c \sigma_1$

Note: # of configs can be infinite.

A word $w \in \Sigma^*$ is accepted
if there is a path in the config
graph

$$(q_0, \varepsilon) \xrightarrow{a_1} (q_1, \sigma_1) \rightarrow \dots$$

$$\xrightarrow{a_n} (q_n, \sigma_n)$$

such that $w = a_1 \dots a_n$

and $q_n \in F$.

A word is acc. on empty-stack if
there is a run with $\sigma_n = \varepsilon$.

Facts.

- Emptiness problem (i.e. $L(A) = \emptyset$) is decidable for PDA. $O(n^3)$.
- Membership is decidable
- PDAs are not closed under complement.
- PDs are not determinizable.

Interprocedural flow analysis

→ PDA reachability/
emptiness.