- Reachability in pushdown automata using games

$$\mathcal{A} = (\Sigma, Q, q_0, \Gamma, \delta, F)$$

finite alphabet

finite set of states

$q_0 \in Q$

finite "stack" alphabet

$F \subseteq Q$

$$\delta \subseteq Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \times Q \times \Gamma_\varepsilon$$

$(q, a, b, q', c) \in \delta$ means

"PDA can go from q to q' reading 'a', popping 'b' & pushing 'c'.

<u>Problem</u>. Is $\mathcal{L}(A)$, defined using empty-stack reachability, nonempty?

$$\mathcal{L}(A) \neq \emptyset ?$$

Games

Reachability game: (V_0, V_1, E, T, v_0)

V_0, V_1: finite disjoint sets

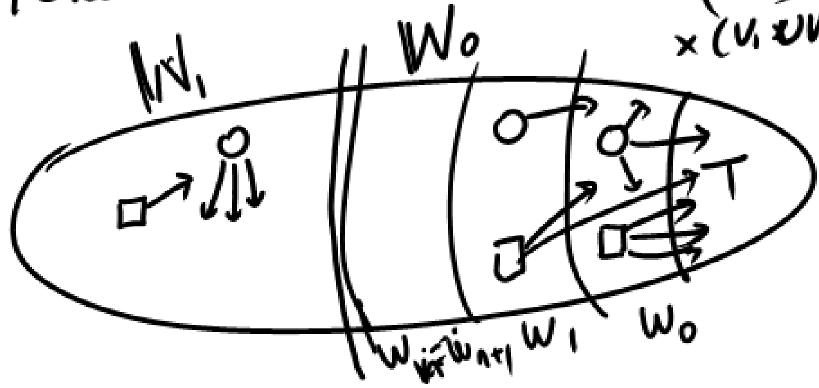
$E \subseteq (V_1 \cup V_2) \times (V_1 \cup V_2)$

$T \subseteq V_0 \cup V_1$

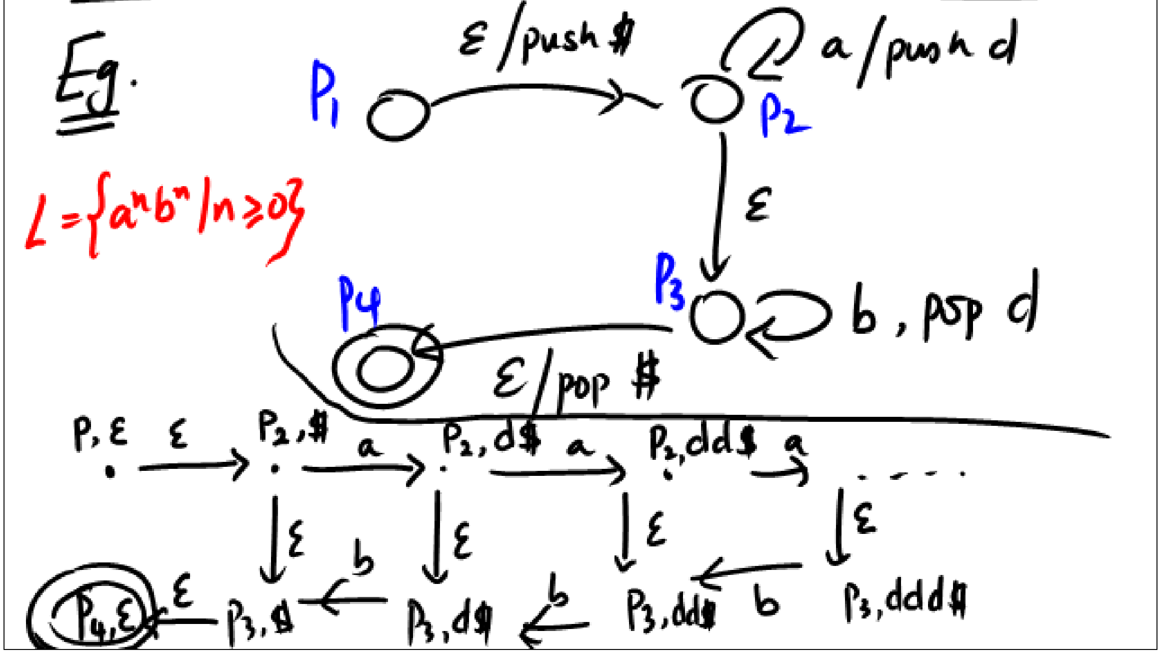
$v_0 \in V_0 \cup V_1$

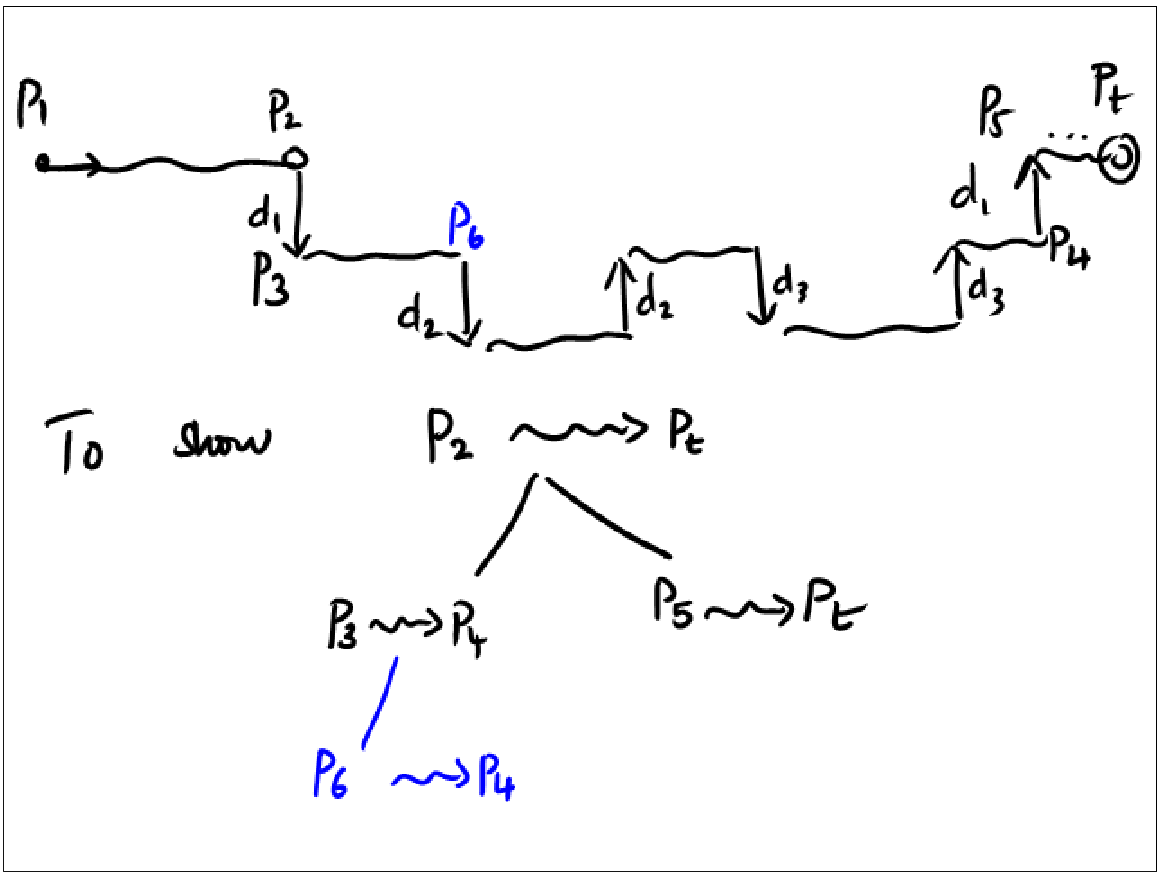
○ — 0-node

□ — 1-node



Reducing PDA to solving reachability games




P1
P2
d1
P3
P6
d2
d2
d3
d3
d1
P4
P5
Pt
To show P2 ~~> Pt
P3 ~~> P4
P5 ~~> Pt
P6 ~~> P4

States: $(Q \times Q) \cup (Q \times Q \times Q \times Q) \cup \{\text{init}\}$

V_0 : $(Q \times Q) \cup \{\text{init}\}$

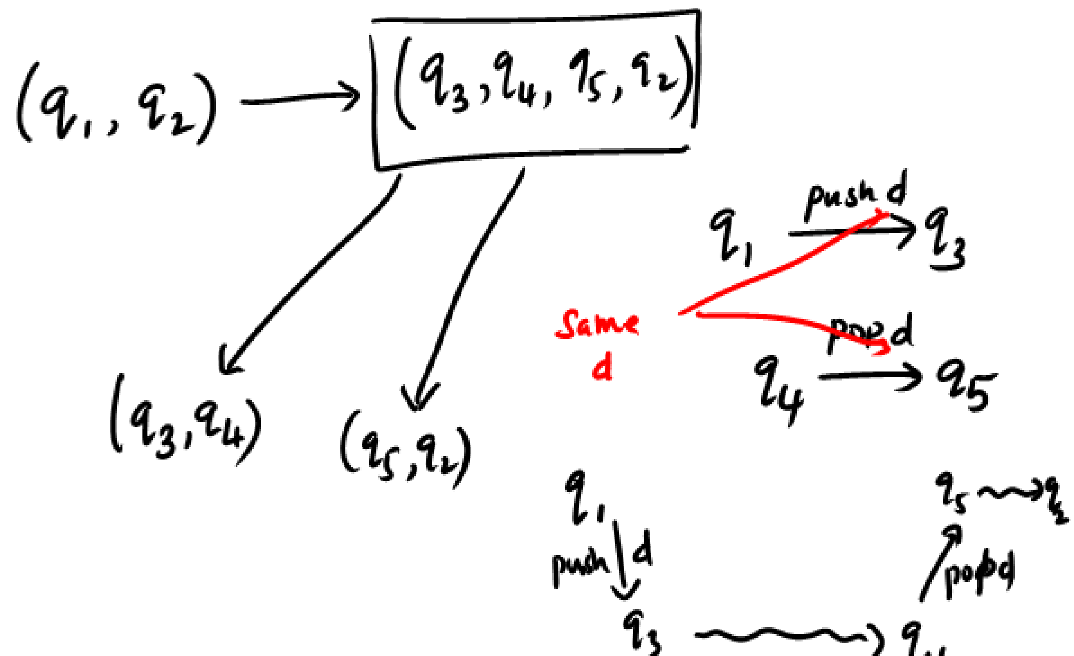
V_1 : $Q \times Q \times Q \times Q$

$\text{init} \longrightarrow (q_0, q_f) \quad q_f \in F$

$$(q_1, q_2) \longrightarrow (q', q_2)$$

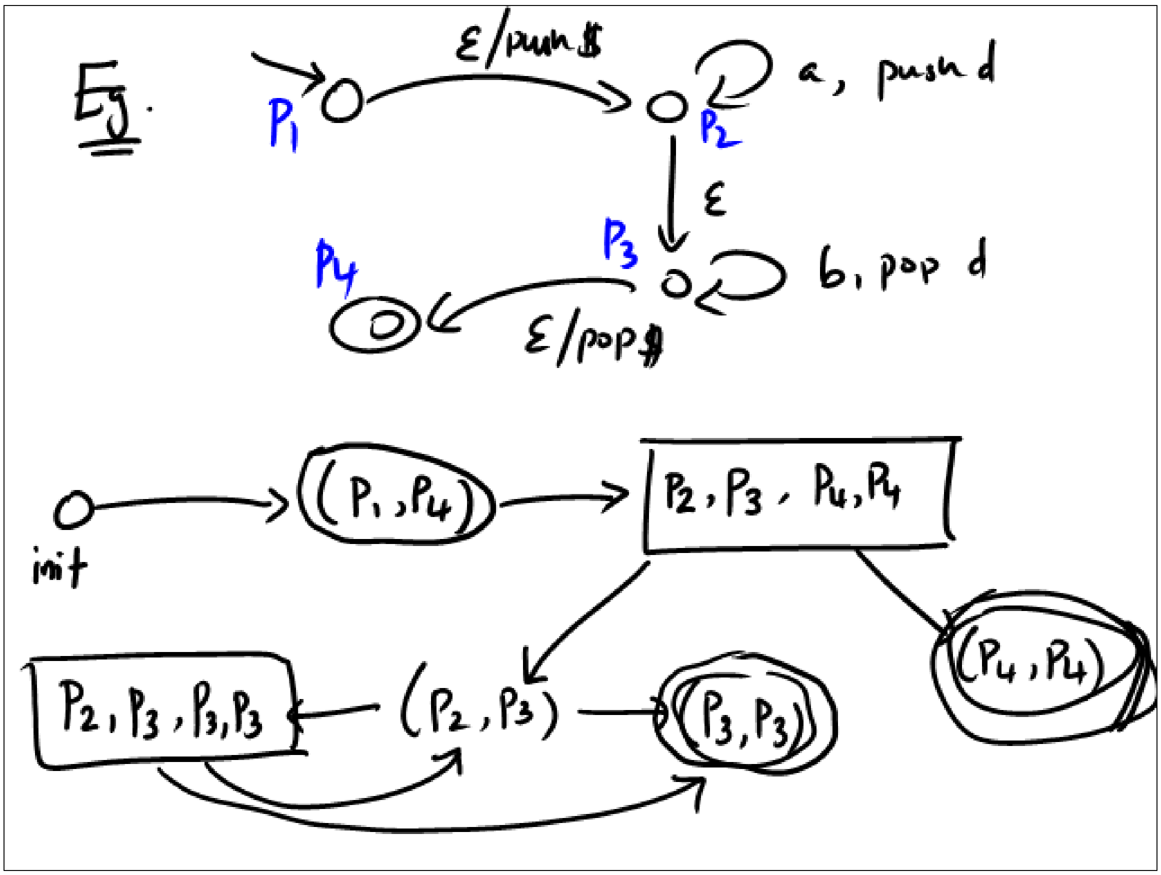
$(q_1, a, \varepsilon, q', \varepsilon) \in \delta$

$q_1 \xrightarrow{a} q'$



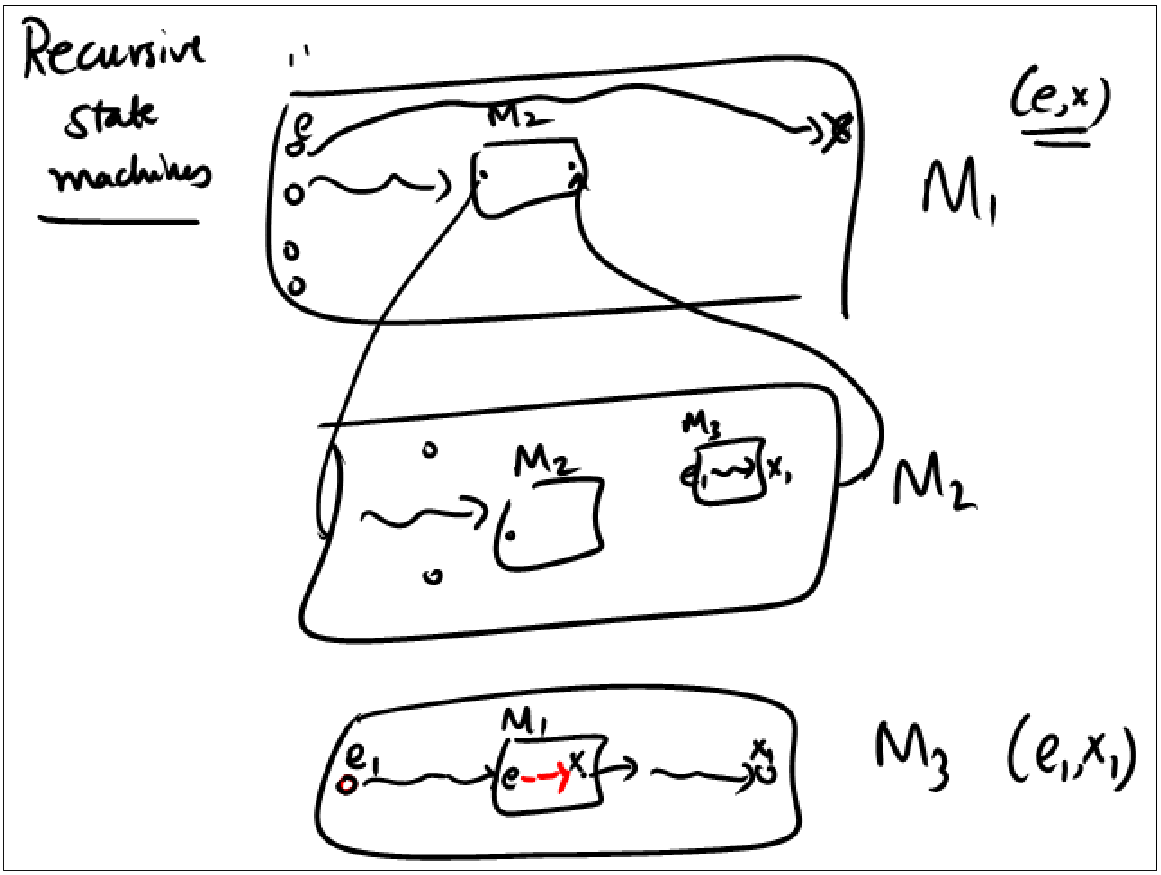
Eve wins if she reaches $(q, q) \mid q \in Q$

Target: $\{(q, q) \mid q \in Q\}$.


Eg.
ε/push $
a, push d
P1
P2
ε
P3
P4
b, pop d
ε/pop $
init
(P1, P4)
P2, P3, P4, P4
(P4, P4)
P2, P3, P3, P3
(P2, P3)
(P3, P3)

PDA reachability reduces $[O(n^3)]$
to a reachability game
on finite graphs.

Hence PDA reachability is solvable
in <u>cubic time</u>.


Recursive state machines
M2
M1
(e,x)
M3
M2
M2
M1
M3 (e1,x1)

RSM reachability.

$$O(n \cdot \theta^2)$$

no. of states in RSM

maximal number of entries/exits in any module.