Interprocedural analysis

& exploiting distributive problems

Def. $(P, D, F, M, \sqcap)$

D: finite set of facts

F: functions $f : 2^D \to 2^D$

M: $M : E \to F$

$\sqcap$: $\cup / \cap$

ϕ, D

$$MOP_n = \prod_{\pi \text{ leading to } n} \left(Pf_\pi(T) \right)$$

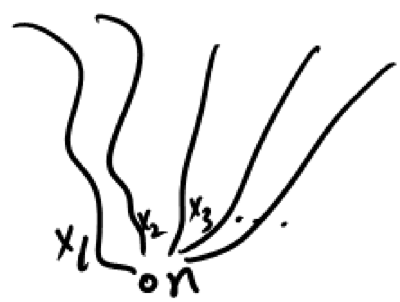
$$\pi = n_0 \xrightarrow{f_0} n_1 \xrightarrow{f_1} n_2 \xrightarrow{f_2} n_3 \cdots \xrightarrow{f_k} n_{k+1}$$

$$pf_\pi(T) = f_k(\ldots f_2(f_1(f_0(T))))$$

Immediate result

We can build a PDA over
states : (program_states × 2^D)
transitions : - mimic the program
- Keeptrack of dataflow facts.

PDA Keeps track of exact dataflow facts computed on any (valid) run.



Is (n, X_1) reachable?

Is (n, X_2) reachable?

$\vdots$

$\forall X_i \subseteq D$

And then compute $\displaystyle\prod_{X_i \mid (n, X_i) \text{ is reachable}} (X_i)$

Exploiting distributivity

$$D \quad f : 2^D \to 2^D$$

$$\sqcap \; - \; \cup \quad ; \quad \top = \emptyset$$

f is *distributive* if

$$f(X \cup Y) = f(X) \cup f(Y).$$

X

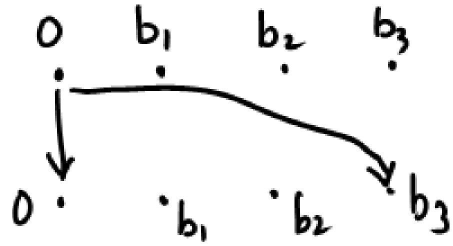
$$f(X) = f(\{a_1\}) \cup f(\{a_2\}) \cup \dots f(\{a_n\})$$

Given a distributive function $f: 2^D \to 2^D$

$$R_f : \{(0,0)\}$$
$$\cup \{(0,y) \mid y \in f(\emptyset)\}$$
$$\cup \{(x,y) \mid y \in f(\{x\}) \ \& \ y \notin f(\emptyset)\}$$

Constant function: $D = \{b_1, b_2, b_3\}$

$f(X) = \{b_3\} \ \forall X \subseteq D.$

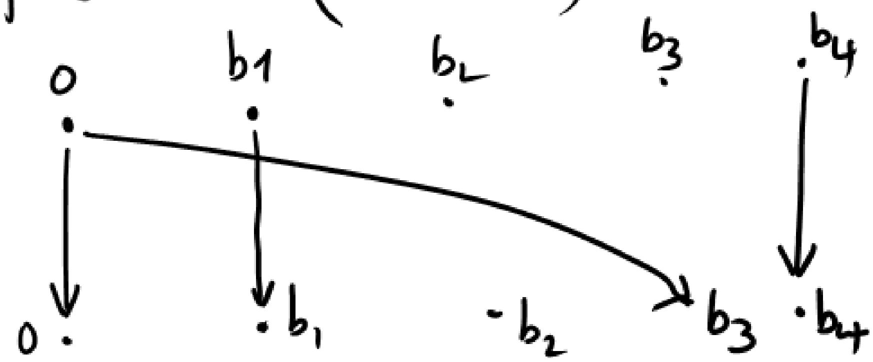


No $b_1 \to b_3$ edge.

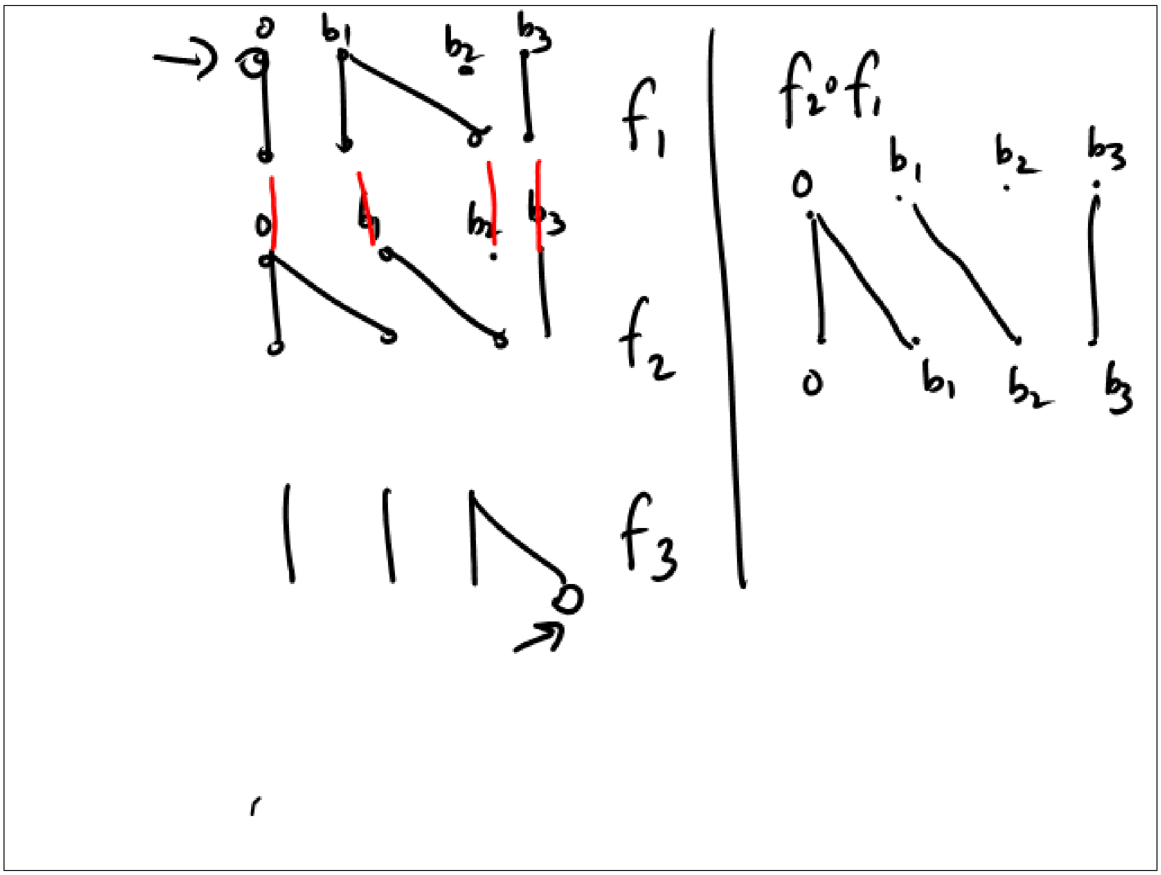
$f(\{b_3\} \cup \emptyset) = f(\{b_3\}) \cup f(\emptyset)$

$$D = \{b_1, b_2, b_3, b_4\}$$

$$f(X) = (X \setminus \{b_2\}) \cup \{b_3\}$$



Lemma. The graph R_f represents f faithfully.


0 b1 b2 b3
f1
0 b1 b2 b3
f2
f3
f2∘f1
0 b1 b2 b3
0 b1 b2 b3

Build PDA with state-space

$$Q \times (\{0\} \cup D)$$

Transitions: $(q, d) \longrightarrow (q', d')$

update q' according to pgm.

update d' according to R_f

$q \xrightarrow{f} q'$

Prop. d holds at n iff (n, d) is reachable

Algorithm.

- Construct PDA
- Compute all reachable states
- $\mathrm{MOP}(n) = \bigcup_{\substack{d \in D \\ (n,d) \text{ is} \\ \text{reachable}}} \{d\}$

Thm: Distributive dataflow analysis is solvable in time $O((|P| \cdot |D|)^3)$