# Modeling Systems

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Lecture #2
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## Modeling systems as graphs

- Kripke structures
- Implicit (symbolic) representation
- First order logic (arithmetic, etc.)
- Modeling FSMs, circuits, games, programs, ...

## Kripke structure

$$M = (AP, S, S_0, R, L)$$

AP: set of Atomic Props.
S: states
$S_0$: initial states
R: $R \subseteq S \times S$ (total)
L: $L : S \to 2^{AP}$

## Implicit representation of graphs

$\wedge$ , $\vee$ , $\Rightarrow$ , $\neg$ , $\exists$ , $\forall$

## Theory of arithmetic

Vocabulary:

Functions: $+, -, *, /$

Predicates: $=, <, >$.

$$\forall x . \exists y . \; x < y$$

– True in arithmetic

$$\forall x \; \exists y . \; (x < y) \wedge (\neg \exists z . \; x < z \wedge z < y)$$

– True in arithmetic.

## Interpreted theories

- Domain
- An interpretation for all functions & predicates.

Symbolic representation of a graph.

$V$ - variables

$S_0$ - A FOL formulas over $V$.

$\varphi_0(V)$

Edges: $R(V, V')$ where $V'$ is a new copy of $V$.

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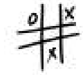
Domain: $-$, x, o, 1, 2




Logic: $=$

$V = \{v_1, \dots, v_9, t\}$

Init: $S_0 : (v_1 = - \wedge v_2 = - \wedge \dots v_9 = -)$
$\wedge (t = 1 \vee t = 2)$

$R(V, V')$.

- $t = 1 \wedge t' = 2 \wedge v_1' = x \wedge \mathrm{Same}(v_2 \dots v_9)$
$\wedge\ v_1 = -$

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$\mathrm{Win}_1 : (v_1 = x \wedge v_5 = x \wedge v_9 = x)$

$\vee$ ⋮

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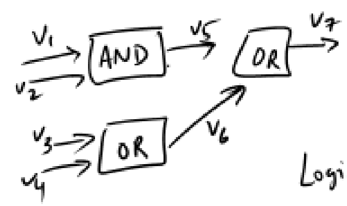


Logic: Boolean
$\wedge, \vee, \Rightarrow, \neg, \dots$

$S_0$ : true

$R(V, V') : v_5' = v_1 \wedge v_2$
$\wedge\ v_6' = v_3 \vee v_4$
$\wedge\ v_7' = v_5 \vee v_6$

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Programs : (integer)

Program $\rightarrow$ Introduce pc's

For every statement, write down the corresponding relation in $R$.

$pc = l$
$x := f(y, x)$
$pc = l'$

$R : (pc = l \wedge pc' = l' \wedge x' = f(y, x))$

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$pc = l$
if $b_1(x, y)$ then $l_1$ else $l_2$
$pc = l'$

In $R$:

$(pc = l \wedge b_1(x, y) \wedge pc' = l_1) \vee$
$(pc = l \wedge \neg b_1(x, y) \wedge pc' = l_2)$

## Loops

$pc = \ell$

while $b(x,y)$ do $\{$ $\quad pc = \ell_1$ $\quad \}$ $\quad pc = \ell_2$

$pc = \ell'$

$$
\begin{aligned}
R:\ & (pc = \ell \wedge b(x,y) \wedge pc' = \ell_1) \\
& \vee (pc = \ell \wedge \neg b(x,y) \wedge pc' = \ell') \\
& \vee (pc = \ell_2 \wedge pc' = \ell)
\end{aligned}
$$

$$
\begin{aligned}
p \Rightarrow q \quad &: \quad \neg p \vee q \\
\neg (p \vee q) \quad &\equiv \quad \neg p \wedge \neg q \\
\neg (p \wedge q) \quad &\equiv \quad \neg p \vee \neg q. \\
\neg \exists x\, \varphi \quad &\equiv \quad \forall x\, \neg(\varphi) \\
\neg \forall x\, \varphi \quad &\equiv \quad \exists x\, \neg(\varphi)
\end{aligned}
$$

$$
\begin{aligned}
\varphi(x, y, pc) : \quad & x = 53 \wedge pc = 2 \\
& \wedge y = x \\
& \wedge \exists z . (2z = y)
\end{aligned}
$$

$S_0(V)$

$R(V, V')$