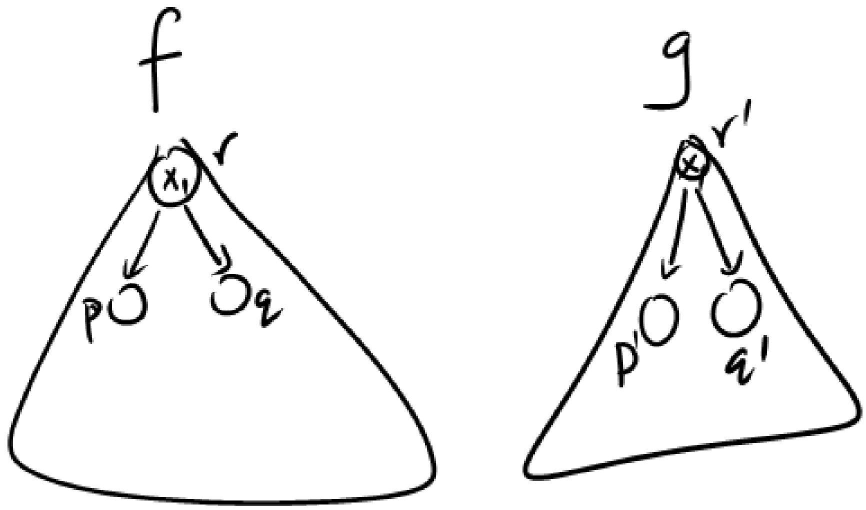
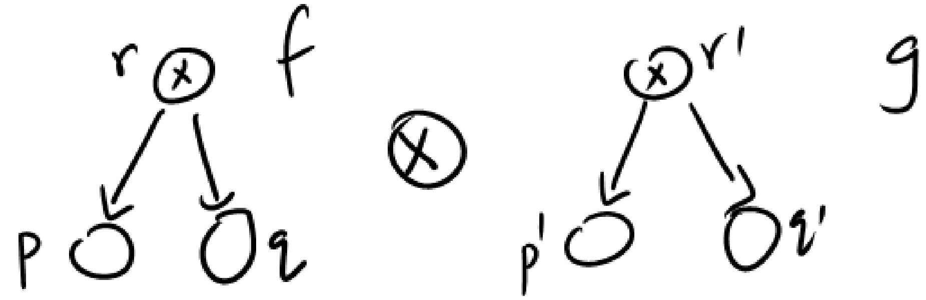
BDDs part 2



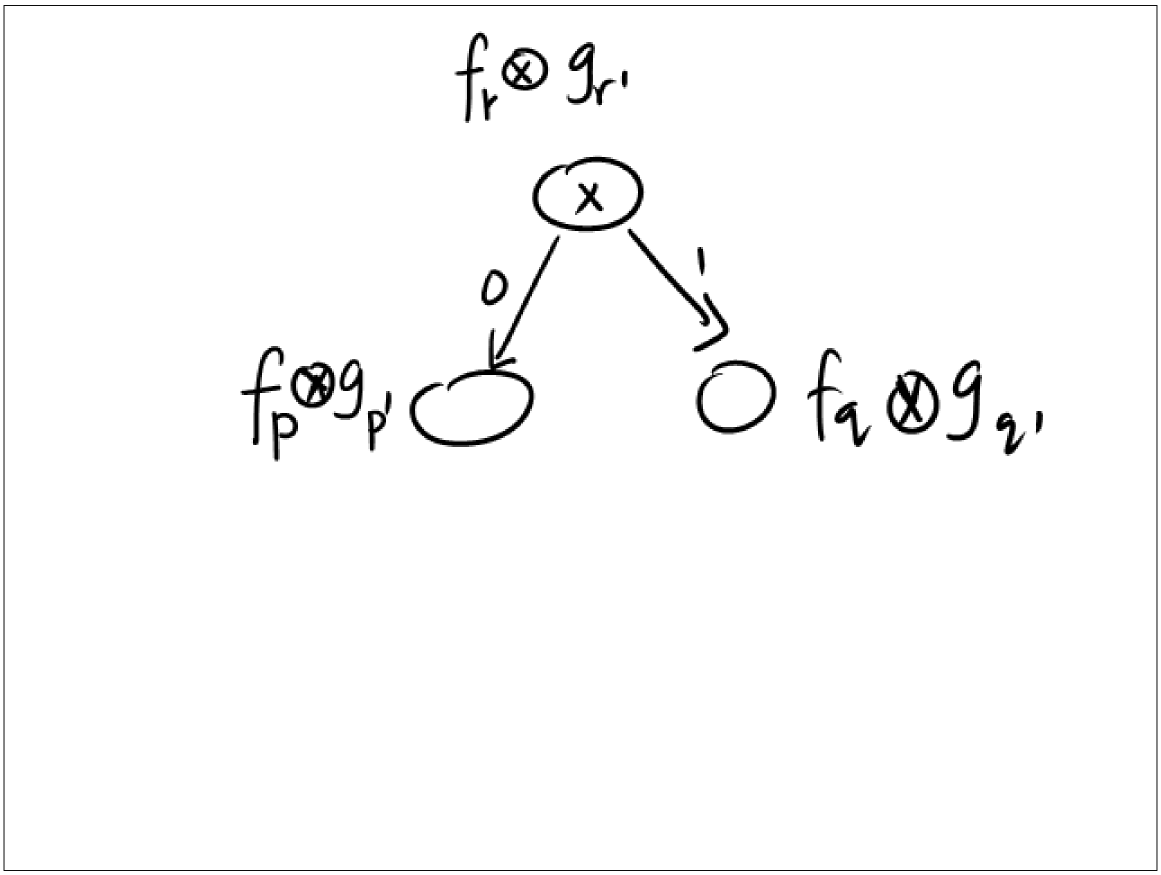
$$f_r = (\neg x_1 \wedge f_p) \vee (x_1 \wedge f_q)$$
$$g_{r'} = (\neg x_1 \wedge g_{p'}) \vee (x_1 \wedge g_{q'})$$



$$(\neg x \wedge f_p) \vee (x \wedge f_q) = f_r$$

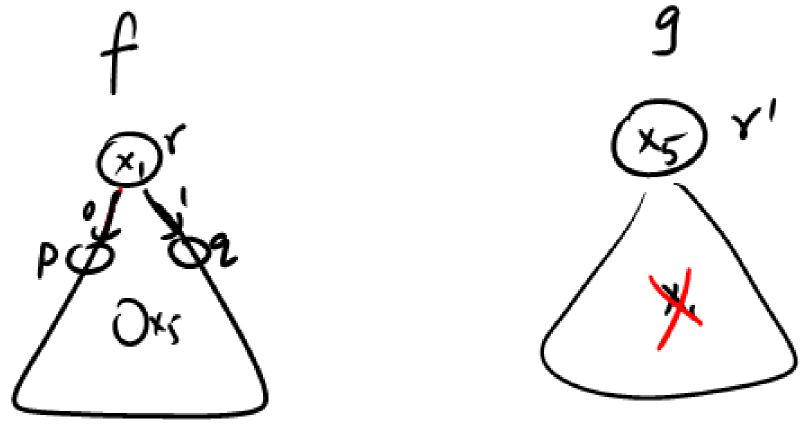
$$(\neg x \wedge g_{p'}) \vee (x \wedge g_{q'}) = g_{r'}$$

$$f_r \otimes g_{r'} : \left[\left[\underbrace{f_p \otimes g_{p'}}_{\text{setting } x = F} \wedge \neg x \right] \vee \left[f_q \otimes g_{q'} \wedge x \right] \right]$$



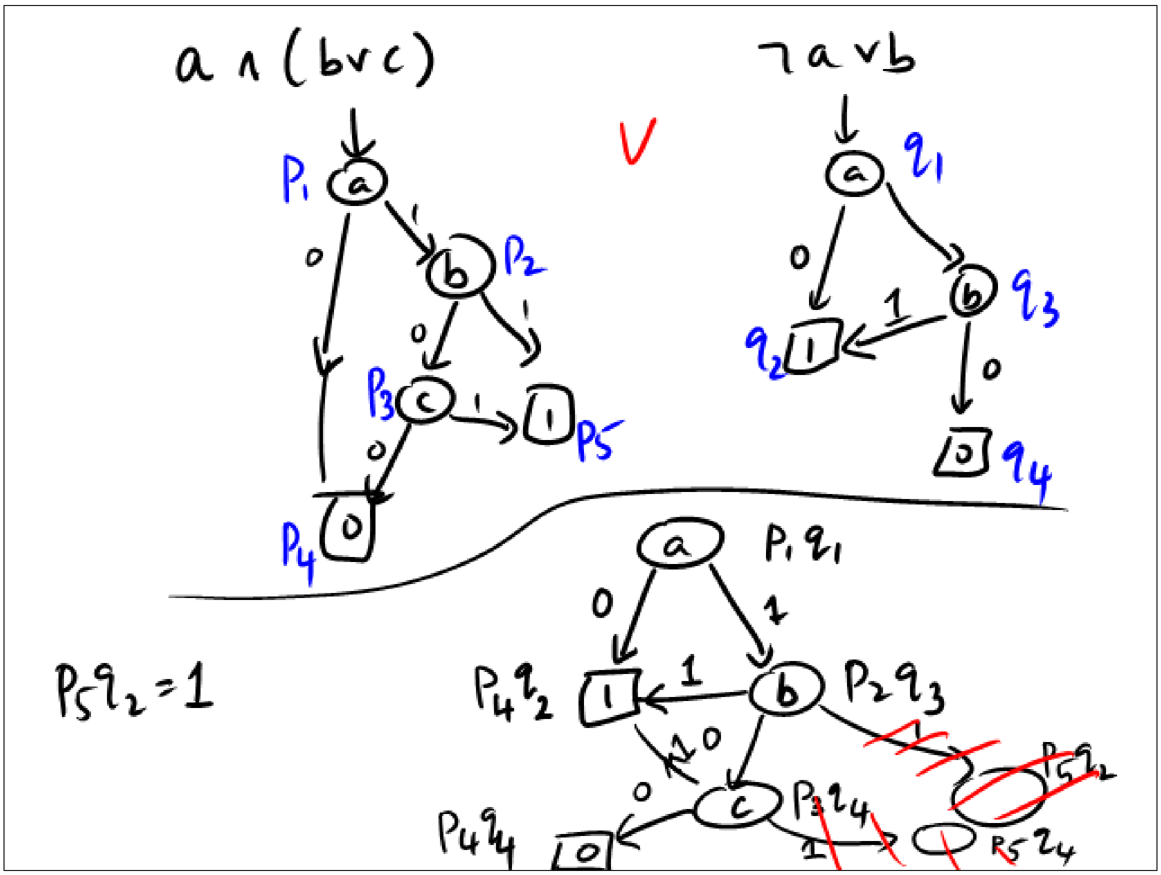
$f_r \otimes g_{r'}$
x
0
1
$f_p \otimes g_{p'}$
$f_q \otimes g_{q'}$

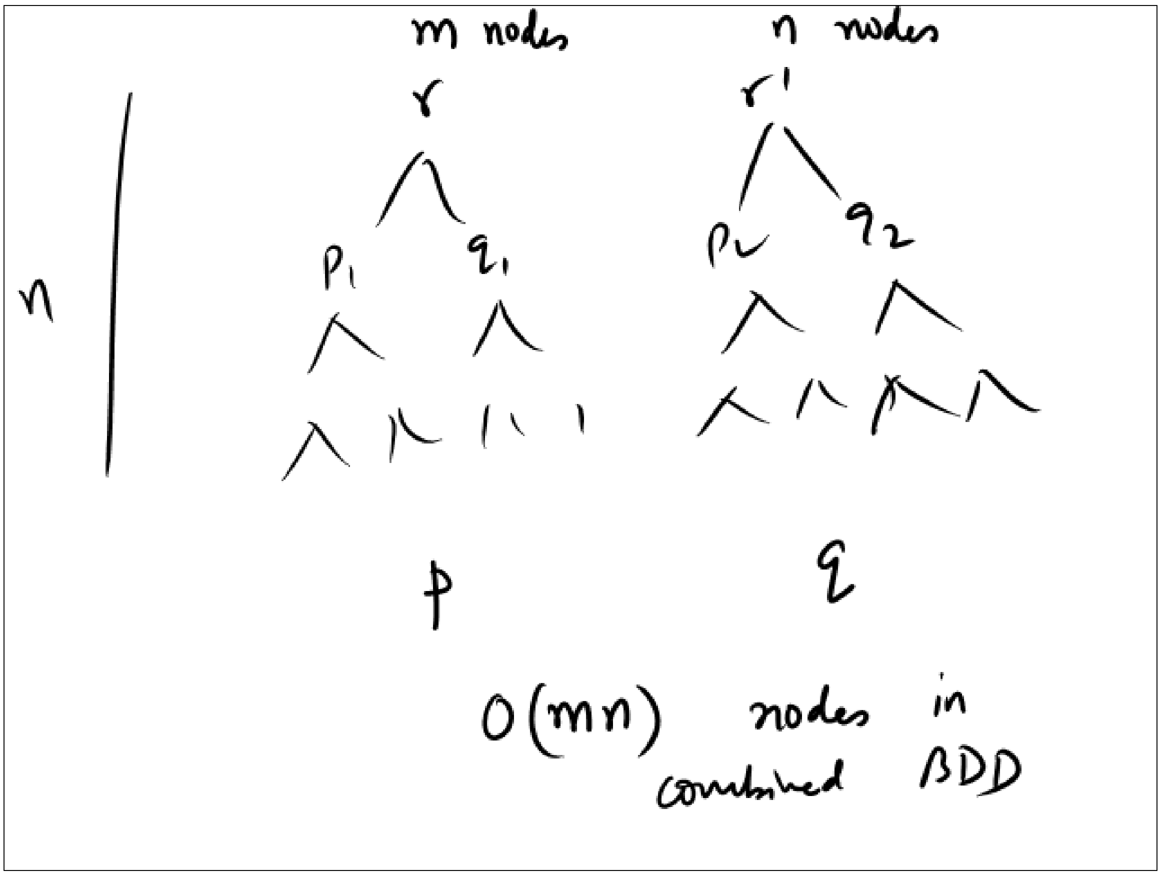
$x_1 < x_5$



$$f_r = (\neg x_1 \wedge f_p) \vee (x_1 \wedge f_q)$$

$$f_r \otimes g_{r'} = (\neg x_1 \wedge (f_p \otimes g_{r'}))$$
$$\vee (x \wedge (f_q \otimes g_{r'}))$$


a ∧ (b∨c)
¬a ∨ b
∨
P1 P2 P3 P4 P5
q1 q2 q3 q4
P1q1
P2q3
P4q2
P3q4
P4q4
P5q2
P5q4
P5q2 = 1


m nodes
n nodes
r
r'
p_1
q_1
p_2
q_2
n
p
q
O(mn) nodes in combined BDD

Thm: Any binary op on BDDs can be done in $O(mn)$ time.

Variable ordering matters

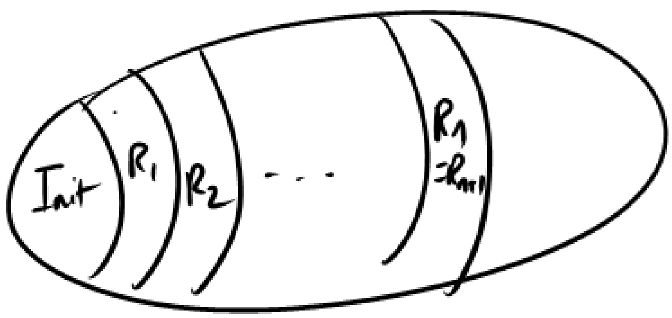
$$f(x_1 \ldots x_n, x_1' \ldots x_n') = \begin{array}{l} x_1 = x_1' \\ \wedge\, x_2 = x_2' \\ \quad \vdots \\ \wedge\, x_n = x_n' \end{array}.$$

$x_1\, x_1'\; x_2\, x_2' \ldots x_n\, x_n'$: order

$O(n)$

$x_1\, x_2 \ldots x_n\; x_1' \ldots x_n'$

2^n nodes necessary



$$R_1(x') = \exists x \left(Init(x) \wedge R(x,x') \right) \vee Init(x)$$

$$R_1(x) = \left[\exists x \; Init(x) \wedge R(x,x') \right]_{x' \to x} \vee Init(x)$$

$$R_2(x) = \left[\exists x \; R_1(x) \wedge R(x,x') \right]_{x' \to x} \vee R_1(x)$$

$R_1, R_2, \ldots R_n, R_{n+1}$

till $R_n = R_{n+1}$

Compute $R_n \wedge F$

Check if $R_n \wedge F = \emptyset$

(by checking if $\boxed{1}$ exists)

$f : S \to S$ S - a set of sets.

$x \in S$ is a fixed-point of f

if $f(x) = x$.

f is monotonic if

$\forall X, Y : X \subseteq Y$

$\Rightarrow f(X) \subseteq f(Y)$

<u>Tarski</u> Any monotonic fn has a lfp.

$$\exists x_i \; f(x_1 \ldots x_n)$$

$$= f(x_1 \ldots x_i/_T, \ldots x_n) \vee f(x_1 \ldots x_i/_F, \ldots x_n)$$

$$= [x_i = T \wedge f(x_1 \ldots x_n)] \vee [x_i = F \wedge f(x_1 \ldots x_n)]$$

$f_{x_i \leftarrow T}$

Tarski

lfp : $\phi, f(\phi), f^2(\phi), \ldots$

If $f : S \longrightarrow S$ is defined

as $f(X) =$ set of states reachable from some state in X by 1 step.

$X \subseteq Y \Rightarrow f(X) \subseteq f(Y)$

Reachability

$$\mathrm{lfp}\, f(x') \left[\mathrm{Init}(x') \vee \exists x' \left(X \vee R(x', x) \right) \right.$$

$\emptyset$

$$f(\emptyset) = \mathrm{Init}$$

$$ff(\emptyset) = \mathrm{Init} \cup \text{Reachable in 1-step from Init}$$

$$\vdots$$

$$f^{n+1}(\emptyset) = f^{n}(\emptyset) = \text{Reachable states.}$$