CTL model checking,

games,

μ-calculus

CTL$_V$ ::= $\underset{(p \in V)}{p}$ | $\varphi \vee \varphi'$ | $\neg\varphi$ |
EX φ | AX φ | EF φ | AF φ |
EG φ | AG φ | E φ U φ' | A φ U φ'

$TS = (S, S_{in}, \rightarrow, Lab)$

$Lab : S \rightarrow 2^V$

$[\![\varphi]\!] = \{s \in S \mid s \models \varphi\}$ inductively

$[\![p]\!] = \{s \in S \mid Lab(s) \ni p\}$

$[\![\varphi \vee \varphi']\!] = [\![\varphi]\!] \cup [\![\varphi']\!] \ \ldots$

CTL MC:

Inductively computing $[\![\varphi]\!]$.

# of subformulas of φ: linear in φ

Each computation is linear in $|TS|$.

So total time $O(|\varphi| . |TS|)$

$\varphi(X, Y)$: $\exists z . [R(x, z) \wedge z \in Y]$

$\varphi : 2^S \longrightarrow 2^S$

$\varphi(T) = \{$ val. of X u $\mid \varphi(u, T)$ is true$\}$

Monotonic $\forall T, T'$:

$T \subseteq T' \Rightarrow \varphi(T) \subseteq \varphi(T')$

If φ is monotonic, then φ has a fixed point.

(φ has a unique minimal fixpoint).

T is a fixed point of f: $f(T) = T$

Reach(p)

$f : T \longmapsto$ set of all states that sat. p $\cup$ set of states that can reach T in one step.

lfp f = Reach(p).

Modal μ-calculus.

$p \mid Z \mid \varphi \vee \varphi \mid \neg\varphi \mid EX\varphi \mid \mu Z\varphi \mid \nu Z\varphi$

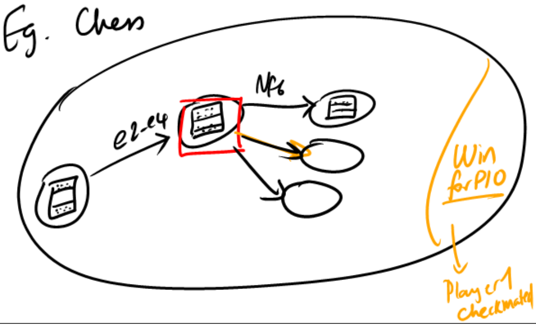
$\mu Y(p \vee EXY)$ - Reachability

CTL: $EGp = \nu Y(p \wedge EXY)$

$EFp = \mu Y(p \vee EXY)$

GAMES

Eg. Chess



Reachability game:

Player 0 wins a play (path) if path reaches a Target state

Player 1 wins a play if path never reaches a Target state

Determinacy: Either player 0 or player 1 has a winning strategy.

Strategy.

For player 0

str_0 : Plays ending in player 0 node $\longrightarrow$ Nodes

Such that $str_0(q_0 q_1 \dots q_n) \longmapsto p$ then $q_n \longrightarrow p$ must be in the graph

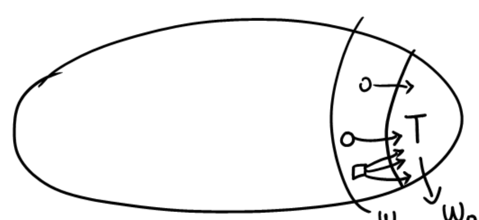
Strategy is zero memory if $str_0(q_0 \dots q_n)$ depends only on q_n.

Zero-memory determinacy

Either player 0 or player 1 has a ws, and in fact winning player has a 0-mem ws.

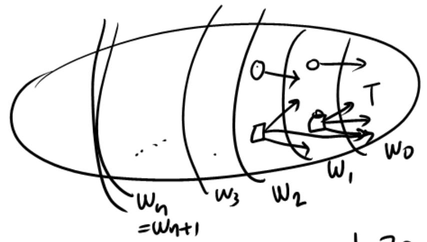
Graph : (V_0, V_1, E, T) V_0 ○ V_1 □

$T \subseteq V_0 \cup V_1$



$W_1 = \{pl\ 0 \text{ nodes } u : \exists v\ u \to v, v \in T\}$

$\cup \{pl\ 1 \text{ nodes } u : \forall v : u \to v \Rightarrow v \in T\}$



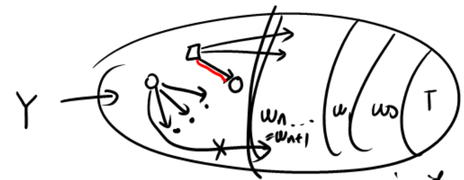
$W_{i+1} = \{$ pl. 0 nodes $u \mid \exists v : u \to v$ and $v \in W_i \}$

$\cup \{$ pl 1 nodes $u \mid \forall v : u \to v \Rightarrow v \in W_i \}$

W_n is fp

Clearly W_n is wining for pl. 0.

Also, player 0 has a zero-memory winning strat. from all nodes in W_n.



All player 0 moves keep me inside Y

There is always a player 1 move that keeps play inside Y.

Hence. for any game

(V_0, V_1, E, T)

$\exists$ W_0, W_1

$W_0 \cup W_1 = V_0 \cup V_1$

$W_0 \cap W_1 = \emptyset$

player0 has a zero mem ws on W_0

and player 1 has a zero mem ws on W_1