Lecture #7

Automata on Infinite Words

&

Linear Temporal Logic

(LTL)

$$\mathcal{A} = (Q, q_0, \delta, F)$$

$$\delta \subseteq Q \times \Sigma \times Q.$$

Run of A on $\alpha \in \Sigma^\omega$

$$\alpha = a_0 a_1 \ldots\ldots$$

$$\rho = q_0 q_1 \ldots\ldots\ldots$$

s.t. $\forall i \quad q_i \xrightarrow{a_i} q_{i+1}$ in A.

ρ is <u>accepting</u> if $\inf(\rho) \cap F \neq \phi$

α is accepted by A if there is some acc run on α.

The language accepted by A,

$$L(A) = \left\{ \alpha \in \Sigma^\omega \,\middle|\, \alpha \text{ is accepted by } A \right\}$$

<u>Def</u>. $L \subseteq \Sigma^\omega$ is <u>ω-regular</u> if there is some Büchi automaton that accepts L.

Language-theoretic properties of ω-regular languages

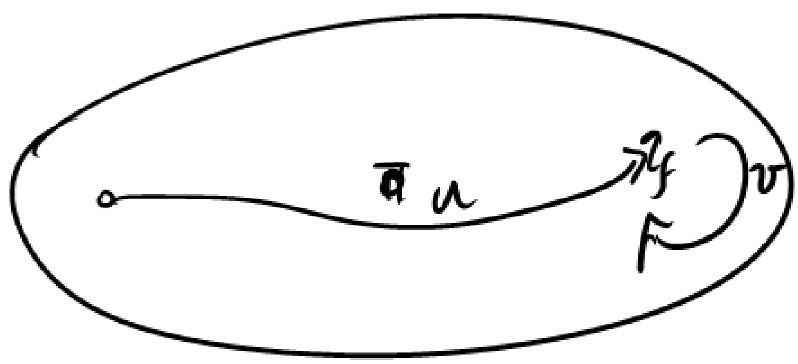
$L_1 \cup L_2$: Closed under union.
(construction is similar to NFA unions)

$L_1 \cap L_2$: Closed under intersection.
Keep track of boolean counter.

Complement. (Büchi '69)

If L is ω-regular, then $\overline{L} = \Sigma^\omega \setminus L$ is also ω-regular.

Emptiness of Büchi automata is decidable



$uv^\omega \in L(A)$

Emptiness decidable in linear time.

Linear Temporal Logic

$$\overset{X_1}{\circ} \longrightarrow \overset{X_2}{\circ} \longrightarrow \overset{X_3}{\circ} \longrightarrow \circ \longrightarrow \circ \longrightarrow \circ \longrightarrow \cdots$$

$(\mathbb{N}, succ)$ $\qquad$ $X_i \subseteq AP$

AP – some finite set.

$$LTL_{AP}: \underset{p \in AP}{p} \;\Big|\; \neg\alpha \mid \alpha \vee \beta \mid X\alpha \mid F\alpha \mid G\alpha \mid \alpha U \beta$$

$$LTL_{AP} = p \mid \alpha \vee \beta \mid \neg\alpha \mid X\alpha \mid F\alpha \mid G\alpha \mid \alpha U \beta .$$

<u>Semantics</u>

$X_1 \quad X_2 \quad \ldots$

$$\sigma \in \left(2^{AP}\right)^{\omega} \qquad X_i \subseteq AP$$

$\sigma, i \models p$ iff $\sigma[i] \ni p$

$\sigma, i \models \alpha \vee \beta$ iff $\sigma[i] \models \alpha$ or $\sigma[i] \models \beta$

$\sigma, i \models \neg\alpha$ iff $\sigma[i] \not\models \alpha$

$\sigma, i \models X\alpha$ iff $\sigma[i+1] \models \alpha$

$\sigma, i \models G\alpha$ iff $\forall j \geq i \,.\, \sigma, j \models \alpha$

$\sigma, i \models \alpha \, \mathcal{U} \, \beta$ iff $\exists j \geq i .$

$\big(\sigma, j \models \beta$ and

$\forall k : (i \leq k < j)$

$\sigma, k \models \alpha \big)$

Every req. is followed by a grant

eventually grant : F grant

$$G\ (\text{request} \Rightarrow (F\ \text{grant}))$$

Grant happens infinitely often

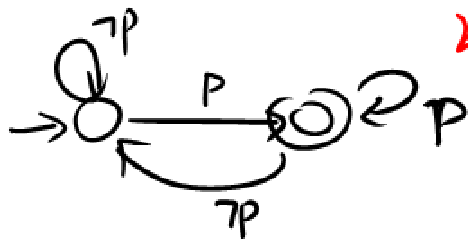
$$GF\ \text{grant}$$

$FG\,\alpha \equiv \alpha$ is eventually an invariant.

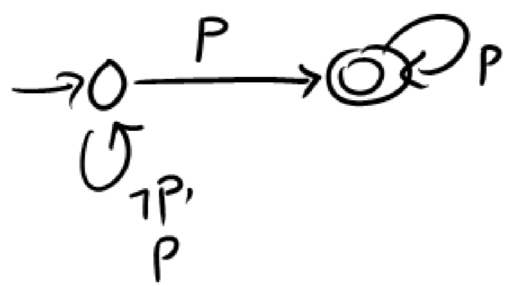
Finitely often α holds $FG\,\neg\alpha$

Det.

GFp :



$FG\,p$:



Not determinizable Büchi

<u>Next class</u>.

LTL $\longrightarrow$ Büchi automata.

$\varphi \xrightarrow{\text{Vardi-Wolper}} A_\varphi$

$|A_\varphi| = 2^{O(|\varphi|)}$.